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⑥ ON THE EXISTENCE OF INFINITELY MANY SOLUTIONS OF THE DIRICHLET PROBLEM FOR SOME NONLINEAR ELLIPTIC EQUATIONS.

⑩ Guang-Chang/Dong Shujie/Li

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Mathematics Research Center  
University of Wisconsin-Madison  
610 Walnut Street  
Madison, Wisconsin 53706

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ON THE EXISTENCE OF INFINITELY MANY SOLUTIONS OF THE  
DIRICHLET PROBLEM FOR SOME NONLINEAR ELLIPTIC EQUATIONS

Guang-Chang Dong<sup>†</sup> and Shujie Li<sup>††</sup>

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ABSTRACT

This paper is concerned with an existence and multiplicity result for non-linear elliptic equations of the type  $Lu = g(x,u) + h(x,u)$  in  $\Omega$ ,  $u = 0$  on  $\partial\Omega$ .  $\Omega \subset \mathbb{R}^N$  is smooth and bounded.  $L$  is a second order self-adjoint, uniformly elliptic operator and the function  $g; \Omega \times \mathbb{R} \rightarrow \mathbb{R}$  is odd with respect to  $u$  and grows like  $f(x)|u|^{p-1}u$ ; the function  $h(x,u) = o(|u|^q)$  where  $n, p$  and  $q$  satisfy certain inequalities.

AMS (MOS) Subject Classifications: 35A15, 35D05, 35J25, 35J60, 58E05.

Key Words: Contractible, critical point, genus, weak solution.

Work Unit Number 1 - Applied Analysis

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<sup>†</sup>Department of Mathematics, Zhejiang University, Hangzhou, China and Mathematics Research Center, University of Wisconsin-Madison.

<sup>††</sup>Institute of Mathematics, Academia Sinica, Peking, China and Mathematics Research Center, University of Wisconsin-Madison.

# SIGNIFICANCE AND EXPLANATION

✓  
The existence of multiple solutions to nonlinear elliptic boundary value problems has been studied by many authors, especially when the nonlinear term is an odd function of the dependent variable. This paper shows, for a class of such equations, that when oddness is destroyed by adding a nonodd nonlinear perturbation to the equation, the resulting problem still possesses an infinite number of distinct solutions.

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ON THE EXISTENCE OF INFINITELY MANY SOLUTIONS OF THE  
DIRICHLET PROBLEM FOR SOME NONLINEAR ELLIPTIC EQUATIONS

Guang-Chang Dong<sup>†</sup> and Shujie Li<sup>††</sup>

In this paper we study the boundary value problem

$$\begin{cases} Lu = g(x, u) + h(x, u) \\ u|_{\partial\Omega} = 0 \end{cases} \quad (1)$$

Let  $\Omega \subset \mathbb{R}^n$  ( $n > 1$ ) be a smooth and bounded domain.  $L$  is a second order self adjoint, uniformly elliptic operator

$$L = - \sum_{i,j=1}^n \frac{\partial}{\partial x_i} (a_{ij}(x) \frac{\partial}{\partial x_j})$$

with  $a_{ij} = a_{ji} \in C^1(\bar{\Omega})$  and

$$\sum_{i,j=1}^n a_{ij}(x) \xi_i \xi_j \geq \eta \sum_{i=1}^n \xi_i^2 \quad \forall x \in \bar{\Omega}, \quad \forall \xi \in \mathbb{R}^n \quad (2)$$

$\eta > 0$  being the ellipticity constant.

$g, h$  are given functions. We assume that

$h : \bar{\Omega} \times \mathbb{R} \rightarrow \mathbb{R}$  is a continuous function and  $h = h(x, s)$  is  $C^1$

with respect to  $s \in \mathbb{R}$  for all  $x \in \bar{\Omega}$ . Moreover

$$h(x, s) = O(|s|^q), \quad h_s(x, s) = O(1) \quad (3)$$

uniformly with respect to  $x \in \bar{\Omega}$  when  $|s|$  is large;  $0 \leq q < 1$ .

We assume that

$g : \bar{\Omega} \times \mathbb{R} \rightarrow \mathbb{R}$  is a continuous function;  $g = g(x, s)$  is

$C^1$  and odd with respect to  $s \in \mathbb{R}$ , for all  $x \in \bar{\Omega}$ .

<sup>†</sup>Department of Mathematics, Zhejiang University, Hangzhou, China and Mathematics Research Center, University of Wisconsin-Madison.

<sup>††</sup>Institute of Mathematics, Academia Sinica, Peking, China and Mathematics Research Center, University of Wisconsin-Madison.

$$g'_s(x, s) \approx p g_1(x) |s|^{p-1} \quad (4)$$

$$g_1(x) > 0 \quad \forall x \in \bar{\Omega} \quad (5)$$

the equivalence relation means  $\lim_{s \rightarrow \infty} \frac{g'_s(x, s)}{p |g_1(x)| |s|^{p-1}} = 1$ , the equivalence being uniform with respect to  $x \in \bar{\Omega}$ , where

$$1 < p < p_{n,q} < \frac{n+2}{n-2} \quad (6)$$

and  $p_{n,q}$  is the largest root of

$$n = \frac{2(p+1)(p-q)}{(p-1)(2p+1-q)}^*.$$

The result of this paper is the following:

**Theorem.** Assume that  $g, h$  satisfying the above restrictions, then problem (1) possesses infinitely many distinct solutions.

In the case (1) is odd, that is  $h(x, u) \equiv 0$ , and  $1 < p < \frac{n+2}{n-2}$  if  $n \geq 3$  or  $1 < p < \infty$  if  $n = 2$ , it is well known that (1) has infinitely many distinct solutions, this follows easily from the well known Lusternik-Schnirelman theory, see Rabinowitz [1]. When  $h$  is independent of  $u$ , i.e.,  $h = h(x)$  and  $1 < p < p_{n,0}$ , (1) possesses infinitely many distinct solutions also, see Bahri and Berestycki [2], [3]. In this paper we use some of the same methods as in [2], [3], but we prove a more general result. Struwe [4], [5] has obtained a result similar to ours but with more restrictive conditions on  $g(x, u)$ .

We express our gratitude to Professors P. Rabinowitz, K. C. Chang, and J. H. Sylvester for many useful conversations.

Before proving the theorem we introduce some notations and lemmas.

\* Let  $f(p) = \frac{2(p+1)(p-q)}{(p-1)(2p+1-q)} - n$ , when  $0 \leq q < 1$ , we have  $f(1+0) = +\infty$ ,  $f(\frac{n+2}{n-2}) < 0$ , hence there exists  $p_{n,q}$  satisfying  $1 < p_{n,q} < \frac{n+2}{n-2}$ ; when  $1 \leq q < p$ ,  $f(p) = 2 \frac{p+1}{(p+1)+(p-q)} \frac{p-q}{p-1} - n < 2 - n < 0$ , i.e., in this case we can not find  $p$  satisfying (6). Hence we must restrict  $q$  to  $0 \leq q < 1$  such that there exists  $p$  satisfying (6).

Let  $H$  be a real Hilbert space with norm  $\|\cdot\|$ . Let  $S = \{x \in H; \|x\| = 1\}$  denote the unit sphere. We consider a functional  $J \in C^0(S, \mathbb{R})$ . For  $a \in \mathbb{R}$  we denote

$$J_a = \{x \in S; J(x) \leq a\}$$

$$\tilde{J}_a = \{x \in S; J(x) \geq a\}.$$

Assume that there exists a constant  $M \in \mathbb{R}$  such that  $J \in C^2(\tilde{J}_M, \mathbb{R})$ . Assume also that  $J$  satisfies the following Palais-Smale condition

(P.S)<sub>M</sub> : For any  $M_1 \geq M$  and for any sequence  $(x_n) \in S$  such that  $M \leq J(x_n) \leq M_1$  and  $\|J'(x_n)\| \rightarrow 0$ , one can extract from  $(x_n)$  a convergent subsequence.

Proposition 1. Suppose  $H$  is infinite dimensional and  $J \in C^0(S, \mathbb{R}) \cap C^2(\tilde{J}_M, \mathbb{R})$  satisfies condition (P.S)<sub>M</sub>. If  $a \in \mathbb{R}$ ,  $a > M$  exists such that  $J_a$  is not contractible in itself to a point, then  $J$  has a critical value in  $[a, +\infty)$ .

Proof: see [3].

Consider the following class of compact symmetric subsets of  $S$

$$M_k = \{A \subset S; A = g(S^k) \text{ where } g \text{ is odd and continuous}\} \quad (8)$$

where  $k \in \mathbb{N}$  and  $S^k = \{x \in \mathbb{R}^{k+1}, |x| = 1\}$  is the  $k$ -dimensional sphere. Let  $J^* \in C^0(S, \mathbb{R}) \cap C^1(\tilde{J}_M^*, \mathbb{R})$  be an even functional. (The superscript  $*$  will always be associated with evenness thereafter.) Define

$$C_k = \inf_{A \in M_k} \max_{x \in A} J^*(x) \quad (9)$$

then we have

Proposition 2. Let  $J^* \in C^1(S, \mathbb{R})$  be even satisfying condition (P.S)<sub>M</sub> and bounded from below on  $S$ . Let  $C_k$  be defined by (9) then:

- (i)  $C_k$  is a critical value of  $J^* \forall C_k \geq M$
  - (ii)  $-\infty < C_k \leq C_{k+1} \forall C_k \geq M$ .
- (10)

Proof. See [3].

Proposition 3. Let  $J \in C^0(S, \mathbb{R}) \cap C^2(\tilde{J}_M, \mathbb{R})$ ,  $J^* \in C^0(S, \mathbb{R}) \cap C^1(\tilde{J}_M^*, \mathbb{R})$  be two functionals both satisfying condition (P.S)<sub>M</sub>. Assume furthermore that  $J^*$  is even and

bounded from below on  $S$ . Let  $C_k$  ( $k \in \mathbb{N}$ ) be defined as in (9). Suppose there exists  $k \in \mathbb{N}$ ,  $\epsilon > 0$  and  $a \in \mathbb{R}$  such that  $C_k > M$  and

$$J_{C_k + \epsilon}^* \subset J_a \subset J_{C_{k+1} - \epsilon}^* \quad (11)$$

then  $J$  has at least one critical value in  $[a, +\infty)$ .

Proof: See [3].

Henceforth we will be working in the space  $H = H_0^1(\Omega)$  with the norm

$$\|u\|^2 = \int_{\Omega} \sum_{i,j=1}^n a_{ij}(x) \frac{\partial u}{\partial x_i} \frac{\partial u}{\partial x_j} \quad (12)$$

$$S = \{u \in H_0^1(\Omega); \|u\| = 1\}.$$

Define for  $u \in H_0^1(\Omega)$

$$I(u) = \frac{1}{2} \|u\|^2 - \int_{\Omega} G(x, u) - \int_{\Omega} H(x, u) \quad (13)$$

$$I^*(u) = \frac{1}{2} \|u\|^2 - \int_{\Omega} G(x, u) \quad (14)$$

( $I^*$  is the even principal part of  $I$ ) where

$$G(x, s) = \int_0^s g(x, t) dt, \quad H(x, s) = \int_0^s h(x, t) dt. \quad (15)$$

From (3), (4) we have

$$H(x, s) \approx O(|s|^{q+1}) \quad (16)$$

$$g(x, s) \approx g_1(x) |s|^{p-1} s \quad (17)$$

$$G(x, s) \approx \frac{g_1(x)}{p+1} |s|^{p+1}. \quad (18)$$

Relations (16), (17), (18) are uniform with respect to  $x \in \bar{\Omega}$ .

Let

$$J(v) = \max_{\lambda \geq 0} I(\lambda v), \quad J^*(v) = \max_{\lambda \geq 0} I^*(\lambda v) \quad \forall v \in S. \quad (19)$$



At a point  $\lambda$  such that  $J(v) = I(\lambda v)$ , one has

$$I(\lambda v) = \frac{\lambda^2}{2} - \int_{\Omega} G(x, \lambda v) - \int_{\Omega} H(x, \lambda v) \quad (20)$$

$$\frac{d}{d\lambda} I(\lambda v) = \lambda - \int_{\Omega} g(x, \lambda v) v - \int_{\Omega} h(x, \lambda v) = 0 \quad (21)$$

$$\frac{d^2}{d\lambda^2} I(\lambda v) = 1 - \int_{\Omega} g'_s(x, \lambda v) v^2 - \int_{\Omega} h'_s(x, \lambda v) v^2 \leq 0 \quad (22)$$

Lemma 1. For any sequence  $v \in S$ ,  $\lambda(v) \in \mathbb{R}^+$  such that  $J(v) = I(\lambda(v)v)$ , the following are equivalent

- (i)  $J(v) \rightarrow +\infty$
- (ii)  $\lambda(v) \rightarrow +\infty$
- (iii)  $v \rightarrow 0$  in  $L^{p+1}(\Omega)$ .

Proof: Since

$$J(v) = I(\lambda(v)v) \quad (23)$$

it is clear that (i)  $\Rightarrow$  (ii).

Suppose that  $\lambda(v) \rightarrow +\infty$ , from (3), (17), (21) we have

$$\lambda(v)^{p-1} \approx \left[ \int_{\Omega} g_1(x) |v|^{p+1} \right]^{-1} \quad (24)$$

From (5) and (24) we have (iii), thus (ii)  $\Rightarrow$  (iii).

Conversely, suppose (iii) is true, we have

$$\int_{\Omega} |v|^{p+1} \rightarrow 0 \Rightarrow \int_{\Omega} v^2 \rightarrow 0 \quad (25)$$

If (ii) is false, i.e.,  $\lambda(v)$  bounded, from (22) we get  $1 \leq 0$ , a contradiction, thus  $\lambda(v) \rightarrow +\infty$ .

Substituting (16), (18) into (20) and using (24) we have

$$J(v) \approx \left( \frac{1}{2} - \frac{1}{p+1} \right) \lambda(v)^2 \approx \left( \frac{1}{2} - \frac{1}{p+1} \right) \left[ \int_{\Omega} g_1(x) |v|^{p+1} \right]^{-\frac{2}{p-1}} \quad (26)$$

Hence  $J(v) \rightarrow +\infty$  this proves the lemma.

Lemma 2. There exists a positive constant  $M > 0$  such that  $J(v) \in C^0(S, \mathbb{R}) \cap C^2(\tilde{J}_M, \mathbb{R})$  where  $\tilde{J}_M = \{v \in S; J(v) \geq M\}$ .

Proof: For fixed  $v \in S$ , by (20), (16), (18) there exists  $\lambda_0 > 0$  such that

$$I(\lambda_0 v) < 0 \text{ as } \lambda > \lambda_0.$$

Therefore the  $\lambda(v)$  satisfying (23) must also satisfy  $0 \leq \lambda(v) < \lambda_0$ . In a small neighborhood of this fixed  $v \in S$  we have

$$J(w) = \max_{\lambda \in [0, \lambda_0]} I(\lambda w), \quad w \in S.$$

But the maximum of a continuous functional on a compact set is continuous, so that

$$J(v) \in C^0(S, \mathbb{R}).$$

By (3), (4), (22) (26) and lemma 1, we have

$$\left[ \frac{d^2}{d\lambda^2} I(\lambda v) \right]_{\lambda=\lambda(v)} = 1 - p + o(1) \text{ as } J(v) \rightarrow +\infty \quad (27)$$

the estimation (27) is uniform for  $x \in \bar{\Omega}$ . Hence a large positive constant  $M$  exists such that

$$\left[ \frac{d^2}{d\lambda^2} I(\lambda v) \right]_{\lambda=\lambda(v)} < 0 \text{ as } J(v) \geq M.$$

Applying the implicit function theorem to  $\frac{d}{d\lambda} I(\lambda v) = 0$  shows that  $\lambda(v) \in C^1(\tilde{J}_M, \mathbb{R})$  and

$$\begin{aligned} \langle J'(v), \phi \rangle &= \lambda(v) \langle I'(\lambda(v)v), \phi \rangle + \langle I'(\lambda(v)v), v \rangle \langle \lambda'(v), \phi \rangle \\ &= \lambda(v) \langle I'(\lambda(v)v), v \rangle \end{aligned} \quad (28)$$

where  $\langle \cdot, \cdot \rangle$  means the duality of  $H^{-1}(\Omega)$  and  $H_0^1(\Omega)$ . (28) is valid in  $H^{-1}(\Omega)$ . Combining (28),  $\lambda(v) \in C^1(\tilde{J}_M, \mathbb{R})$  and  $I(u) \in C^2$  shows that

$$J(v) \in C^2(\tilde{J}_M, \mathbb{R}).$$

Thus the proof of lemma 2 is completed.

Lemma 3.  $J(v)$  satisfies (F.S)<sub>M</sub>.

Proof: Assume that  $(v_n) \in S$  satisfies

$$M \leq J(v_n) \leq M_1 \quad (29)$$

where  $M_1 \geq M$  are constants and

$$\|J'(v_n)\|_{H^{-1}} \rightarrow 0 \quad (n \rightarrow \infty) \quad (30)$$

where  $J'(v_n)$  only operates on  $S$ .

We shall use (26), but in this case the meaning (26) has changed a little because  $M$  is fixed. The meaning of (26) is: having first fixed a positive  $\delta$ , we can select a positive large  $M$  such that both (26) and

$$(1-\delta)J(v) < \left(\frac{1}{2} - \frac{1}{p+1}\right)\lambda^2(v) < (1+\delta)J(v) \quad \forall v \in J_M \quad (31)$$

$$(1-\delta)J(v) < \left(\frac{1}{2} - \frac{1}{p+1}\right) \int_{\Omega} g_1(x) |v|^{p+1} < (1+\delta)J(v) \quad \forall v \in J_M \quad (32)$$

are true; the estimates (31), (32) are uniform for  $x \in \bar{\Omega}$ . Taking  $\delta = \frac{1}{2}$  is sufficient for our use. We prove the lemma with such a fixed  $M$ .

Denote  $\lambda(v_n)v_n = u_n$ . We denote hereafter by  $C$  various positive constants.

By (31)  $\|u_n\| = \lambda(v_n)$  is bounded by  $\left[\frac{2(p+1)}{p-1} (1-\delta)M\right]^{\frac{1}{2}} \leq \|u_n\| \leq \left[\frac{2(p+1)}{p-1} (1+\delta)M_1\right]^{\frac{1}{2}}$ . By (6) the injection  $H_0^1(\Omega) \rightarrow L^p(\Omega)$  is compact. Therefore there exists a subsequence of  $u_n$ , denoted again by  $u_n$ , which converges strongly in  $L^p(\Omega)$ .

For fixed  $\phi \in H_0^1(\Omega)$ , we have the decomposition  $\phi = t_n u_n + \psi_n$ , where  $\psi_n \perp u_n$  and it is easy to see that  $\|\psi_n\| \leq \|\phi\|$ . By (28) we have

$$\begin{aligned} \langle I'(u_n), \phi \rangle &= t_n \langle I'(u_n), u_n \rangle + \langle I'(u_n), \psi_n \rangle = \langle I'(u_n), \psi_n \rangle \\ &= \frac{1}{\lambda(v_n)} \langle J'(v_n), \psi_n \rangle. \end{aligned} \quad (33)$$

By (30), (33)

$$I'(u_n) = - \sum_{i,j=1}^n \frac{\partial}{\partial x_i} (a_{ij}(x) \frac{\partial u_n}{\partial x_j}) - g(x, u_n) - h(x, u_n) \quad (34)$$

converges strongly to zero in  $H^{-1}(\Omega)$ ,  $g(x, u_n) + h(x, u_n)$  converges strongly in  $L^1(\Omega)$  and also in  $H^{-1}(\Omega)$ . By (34) we have  $u_n$  converges strongly in  $H_0^1(\Omega)$ ,  $u_n \rightarrow u$ . Hence  $v_n = \frac{u_n}{\|u_n\|} \rightarrow \frac{u}{\|u\|}$  in  $S$  if  $\|u\| \neq 0$ . But

$$\|u\| = \lim_{n \rightarrow \infty} \|u_n\| = \lim_{n \rightarrow \infty} \lambda(v_n) \geq \left[ \frac{2(p+1)}{p-1} (1-\delta)M \right]^{\frac{1}{2}} > 0.$$

Therefore the proof of lemma 3 is complete.

Lemma 4. There exists constant  $d$  such that

$$|J(v) - J^*(v)| \leq d \min[J(v), J^*(v)]^{\frac{q+1}{p+1}} \text{ as } J(v) \geq M. \quad (35)$$

Proof: Let  $u = \lambda(v)v$  we have

$$J(v) = I(\lambda(v)v) = I^*(\lambda(v)v) - \int_{\Omega} h(x, u)u. \quad (36)$$

From (36) by using (3), (26) we have

$$\begin{aligned} J(v) - J^*(v) &\leq \left| \int_{\Omega} h(x, u)u \right| \leq C \int_{\Omega} |\lambda(v)v|^{q+1} + C \int_{\Omega} |\lambda(v)v| \\ &\leq C \lambda(v)^{q+1} \left[ \int_{\Omega} |v|^{p+1} \right]^{\frac{q+1}{p+1}} + C \int_{\Omega} |\lambda(v)| \leq C \left[ \int_{\Omega} g_1(x) |v|^{p+1} \right]^{\frac{2(q+1)}{p^2-1}} \end{aligned}$$

a similar estimate can be obtained for  $J^*(v) - J(v)$ . Hence

$$|J(v) - J^*(v)| \leq C \left[ \int_{\Omega} g_1(x) |v|^{p+1} \right]^{\frac{2(q+1)}{p^2-1}}. \quad (37)$$

Combining (26) and a similar equivalent relation for  $J^*(v)$  with (37) we obtain (35).

Proposition 4. For  $J^*(v)$  defined by (19). Define  $C_k$  by (9). Then we have

$$C_k \geq \begin{cases} C k^{\frac{n+2-(n-2)p}{n(p-1)}} & (n \geq 3) \\ C k^{\frac{2}{p-1} - \epsilon_1} & (n = 2) \text{ (positive } \epsilon_1 \text{ sufficiently small)} \end{cases} \quad (38)$$

for  $k$  large enough;  $C$  is a positive constant.

Proof: See [3].

For the proof of (38), in [3] it was assumed that  $\exists \mu, s_0, 0 < \mu < 1, s_0 > 0$  such that

$$0 < \frac{g(x,s)}{s} \leq \mu g'_s(x,s) \quad \forall x \in \bar{\Omega}, \forall s \geq s_0$$

this condition is used in the proof of lemmas 7 and 9 in [3], which corresponds to our lemma 1 and (26). We do not need this condition.

We now come to the proof of our theorem. If the conclusion of the theorem were not true, i.e., if the number of solutions of problem (1) were finite, according to proposition 1, 2 and 3, (11) would not be true for any sufficiently large  $a$  and any  $k$ .

Let

$$\theta(t) = dt^{\frac{q+1}{p+1}}$$

where  $d$  is the constant in (35). If

$$C_{k+1} - \epsilon \geq C_k + \epsilon + \theta(C_k + \epsilon) + \theta(C_k + \epsilon + \theta(C_k + \epsilon)) \quad (39)$$

is true, we take  $a = C_k + \epsilon + \theta(C_k + \epsilon)$  by using (35) and (39) we obtain (11). Hence (39) cannot be true for  $k$  large enough. In other words, there exists a  $k_0$  such that

$$C_{k+1} - C_k \leq \theta(C_k + \epsilon) + \theta(C_k + \epsilon + \theta(C_k + \epsilon)) + 2\epsilon \quad \forall k \geq k_0$$

so that

$$\begin{aligned} C_k - C_{k_0} &\leq \sum_{l=k_0}^{k-1} [\theta(C_l + \epsilon) + \theta(C_l + \epsilon + \theta(C_l + \epsilon)) + 2\epsilon] \\ &\leq \sum_{l=k_0}^{k-1} C_l^{\frac{q+1}{p+1}} \leq k C_k^{\frac{q+1}{p+1}}. \end{aligned}$$

Therefore

$$C_k \leq C k^{\frac{q+1}{p+1}}$$

or

$$C_k \leq C k^{\frac{p+1}{p-q}}.$$

Combining with (38) we have

$$\frac{n+2-(n-2)p}{n(p-1)} < \frac{p+1}{p-q} + \varepsilon_1$$

or

$$p \geq p_{n,q} - \varepsilon_1 (\varepsilon_1 = 0 \text{ for } n \geq 3; \text{ for } n = 2, \varepsilon_1 > 0 \text{ and sufficiently small})$$

where  $p_{n,q}$  is defined by (7). Hence (6) is false. Therefore when (6) is true, the number of solutions of problem (1) must be infinite. This proves the theorem.

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This paper is concerned with an existence and multiplicity result for non-linear elliptic equation of the type $Lu = g(x,u) + h(x,u)$ in $\Omega$ , $u = 0$ on $\partial\Omega$ . $\Omega \subset \mathbb{R}^N$ is smooth and bounded. $L$ is a second order self adjoint, uniformly elliptic operator and the function $g; \Omega \times \mathbb{R} \rightarrow \mathbb{R}$ is odd with respect to $u$ and grows like $f(x) u ^{p-1}u$ ; the function $h(x,u) = o( u ^q)$ where $n, p$ and $q$ satisfy certain inequalities.		



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